BEBR

FACULTY WORKING
PAPER NO. 1428

Economies of Scope and Multiproduct Clubs

Jan K. Brueckner
Kangoh Lee

WORKING PAPER SERIES ON THE POLITICAL ECONOMY OF INSTITUTIONS
NO. 7

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

FACULTY WORKING PAPER NO. 1428

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

January 1988

WORKING PAPER SERIES ON THE POLITICAL ECONOMY OF INSTITUTIONS NO. 7

Economies of Scope and Multiproduct Clubs

Jan K. Brueckner, Professor
Department of Economics

Kangoh Lee, Graduate Assistant
Department of Economics

Economies of Scope and Multiproduct Clubs

by

Jan K. Brueckner and Kangoh Lee
Department of Economics
University of Illinois at Urbana-Champaign
Champaign, IL 61820

January 1988

Abstract

This paper addresses the potential optimality of multiproduct clubs when the joint cost function of such clubs exhibits economies of scope. On the one hand, the joint provision of public goods in multiproduct clubs leads to cost savings under economies of scope. However, there is an efficiency loss from such clubs, since consumption group sizes cannot be set individually for each good. In addition to formalizing these costs and benefits and deriving conditions under which multiproduct clubs are desirable, the analysis characterizes the equilibrium club configuration.

Economies of Scope and Multiproduct Clubs

by

Jan K. Brueckner and Kangoh Lee*

1. Introduction

Local public goods in the U.S. are provided by a variety of different governmental units. While the most visible public goods (education and police and fire protection) are provided by school districts and municipalities respectively, other less important goods are frequently provided by special districts. The growth of such districts can be seen in Table 1, which shows the total number of local government units by type in the U.S. in 1952 and 1982.[1] While the decline in the number of school districts (which reflects consolidation) is noteworthy, equally striking is the tripling of the number of special districts between 1952 and 1982. Table 2 shows the types of services provided by single-function special districts in the U.S., giving the number of districts of each type. The most common special districts are those providing fire protection (in sparsely populated areas), housing and community development services, and drainage and flood control.

The creation of a special district reflects an attempt to provide a particular public good to an optimal-size consuming group. With a congested public good, this optimal size is achieved when the marginal benefit from cost sharing equals the marginal congestion cost of accepting an additional person in the consumption group. When the optimal group size for a particular good (parks, for example) does not match the population size of an existing governmental unit, creation of a special (park) district with different boundaries may be desirable.

Although special districts allow great flexibility in the delivery of public services, it is not necessarily efficient for each public good to be provided by a different governmental unit. The reason is that many public goods exhibit cost complementarities that can be exploited when the goods are provided jointly by a single government. These complementarities can arise from savings in fixed costs or from synergistic interaction between the different public production processes. Police and fire protection provide a good example. Sharing of communications facilities by a municipality's police and fire departments leads to fixed cost savings, and the common service area enhances coordination between police and fire units.[2] Joint provision of police and fire protection in most areas appears to be a consequence of these effects.

In deciding whether joint provision of public goods is desirable, the benefits from cost complementarity must be weighed against the efficiency loss inherent in such an arrangement. This loss arises because joint provision makes it impossible to separately adjust consumption group sizes to suit the congestion properties of each public good. When congestability varies significantly across public goods, this efficiency loss can be substantial. If the loss is large enough to outweigh the gain from cost complementarity, then the goods should be provided by separate, optimal-size jurisdictions. Otherwise, the goods should be provided jointly by a multiproduct government.

The purpose of the present paper is to provide a formal analysis of the trade-off between cost complementarity and efficiency loss in multiproduct governments using a variant of the standard model of clubs (see Berglas and Pines (1981)). To capture the notion of cost complementarity, the analysis borrows the concept of economies of scope from the literature on contestable

markets (see Baumol, Panzar, and Willig (1982)). Introduction of this new concept into the theory of clubs is an important contribution of the paper.[3] After discussing the normative question, the paper analyses the equilibrium jurisdiction structure that emerges in the model under the assumption that public goods are provided by profit-maximizing community developers.

2. The Model

For simplicity, assume that all people in the economy are identical in that they have the same preferences and initial endowments. A representative member's preferences are given by the well-behaved utility function $U(z_a,z_b,x)$, where z_a and z_b are consumption of two public goods and x is consumption of a private composite good.

The two public goods may be produced either separately by single-product clubs or jointly by multiproduct clubs. $C(z_a,z_b,n)$ denotes the cost of providing public goods z_a and z_b jointly to a multiproduct club of n people. When z_a and z_b are produced separately by single-product clubs, costs are $C(z_a,0,n_a)$ and $C(0,z_b,n_b)$ respectively, where n_a and n_b are consumption group sizes for type-a and the type-b single-product clubs. Note that the cost of the type-a club, $C(z_a,0,n_a)$, is obtained by setting $z_b = 0$ in the cost function of the multiproduct club and replacing n by n_a, which allows the group size to be chosen to reflect the congestion properties of z_a. Analogous comments apply to $C(0,z_b,n_b)$. Since the cost of multiproduct clubs is $C(z_a,z_b,n)$, such clubs are forced to provide the the two public goods to the same number of people n even if the two goods have different congestion properties. Congestion, of course, implies that C is increasing in n. Per capita cost C/n is in addition assumed to be a U-shaped function of n for $(z_a,z_b) \neq (0,0)$. This guarantees the existence of a finite optimal club size for any single-product or

multiproduct club.

To address the potential optimality of multiproduct clubs, we introduce the concept of economies of scope, which has been widely used in the literature of contestable markets. With economies of scope, cost savings result from joint production of several different outputs in a single firm. To adapt this concept to the club model, it is necessary to consider an additional variable, consumption group size, in characterizing economies of scope. Formally, we define economies of scope for public goods in the two good case as follows:

Definition 1. There are economies of scope at output vector (z_a, z_b) given the consumption group size n if

$$C(z_a, 0, n) + C(0, z_b, n) > C(z_a, z_b, n).$$

Analogously, weak economies of scope exist if the inequality is weak, and diseconomies of scope exist if the inequality is reversed.

3. Single-Product Versus Multiproduct Clubs

In an economy consisting of identical people, an efficient allocation is obtained by maximizing the utility of a representative individual,

$$U(z_a, z_b, x), \tag{1}$$

subject to the appropriate economy-wide resource constraint. This constraint depends upon the form of clubs providing the two public goods z_a and z_b. With multiproduct clubs, the resource constraint is written as

$$Nx + (N/n)C(z_a, z_b, n) = NI, \tag{2}$$

where N is the total population and I is the endowment of a representative individual. The constraint (2) states that total consumption of the private

good and total cost of providing public goods equals the total endowment in the economy. On the other hand, if z_a and z_b are produced separately by single-product clubs, the resource constraint becomes

$$Nx + (N/n_a)C(z_a,0,n_a) + (N/n_b)C(0,z_b,n_b) = NI. \tag{3}$$

Note that the number of clubs in the two cases (N/n in the multiproduct case, N/n_a and N/n_b in the single-product case) need not be integer-valued. As is standard, this problem is ignored.

The optimality conditions for the multiproduct clubs are found by maximizing utility subject to the constraint (2). These conditions include the Samuelson conditions for the public goods,

$$nU_1/U_3 = C_1 \text{ and } nU_2/U_3 = C_2, \tag{4}$$

and the per capita cost-minimization condition

$$C/n = C_3, \tag{5}$$

where $C \equiv C(z_a,z_b,n)$ and subscripts denote partial derivatives. Similarly, the optimality conditions for the type-a single-product clubs include the Samuelson condition

$$n_aU_1/U_3 = C_{a1} \tag{6}$$

and the per capita cost-minimization condition

$$C_a/n_a = C_{a3}, \tag{7}$$

where $C_a \equiv C(z_a,0,n_a)$. Analogous conditions hold for the type-b single-product clubs.

Letting * denote the optimal values of the choice variables, the maximal utilities with multiproduct clubs and single-product clubs are $u^{*m} \equiv U(z_a^{*m}, z_b^{*m}, x^{*m})$ and $u^{*s} \equiv U(z_a^{*s}, z_b^{*s}, x^{*s})$ (superscripts m and s denote multiproduct and single-product clubs respectively). Our task is to compare u^{*m} with u^{*s} to determine which form of clubs is optimal. As a first step, it is useful to ask which form of clubs yields higher utility given some public good levels (z_a, z_b). This question is the same as asking which form of clubs costs less to a representative individual. To see this, note that $u^m \equiv U(z_a, z_b, x^m) > (\leq)\ u^s \equiv U(z_a, z_b, x^s)$ as $x^m > (\leq)\ x^s$ given (z_a, z_b). But from the resource constraints (2) and (3), we have

$$x^m = I - C(z_a, z_b, n)/n \tag{8}$$

$$x^s = I - C(z_a, 0, n_a)/n_a - C(0, z_b, n_b)/n_b. \tag{9}$$

Thus, $x^m > (\leq)\ x^s$ for given n, n_a, and n_b as

$$C(z_a, z_b, n)/n < (\geq)\ C(z_a, 0, n_a)/n_a + C(0, z_b, n_b)/n_b. \tag{10}$$

To assure that costs are as low as possible, the group sizes in (10) must be chosen optimally conditional on z_a and z_b. The consumption group size that minimizes the per capita cost in multiproduct clubs is implicitly defined by the optimality condition (5) as $n = n(z_a, z_b)$. Analogously, the n_a and n_b that minimize per capita costs in type-a and the type-b single-product clubs are defined by the optimality condition (7) and its counterpart as $n_a = n_a(z_a)$ and $n_b = n_b(z_b)$. Thus, with n, n_a, and n_b chosen optimally, the inequality (10) becomes

$$\begin{aligned} &C(z_a, z_b, n(z_a, z_b))/n(z_a, z_b) < (\geq) \\ &\quad C(z_a, 0, n_a(z_a))/n_a(z_a) + C(0, z_b, n_b(z_b))/n_b(z_b). \end{aligned} \tag{11}$$

This inequality shows the trade-off involved in multiproduct clubs. With economies of scope, by definition we have

$$C(z_a,z_b,n(z_a,z_b))/n(z_a,z_b) <$$
$$C(z_a,0,n(z_a,z_b))/n(z_a,z_b) + C(0,z_b,n(z_a,z_b))/n(z_a,z_b).$$

But, we know that

$$C(z_a,0,n(z_a,z_b))/n(z_a,z_b) \geq C(z_a,0,n_a(z_a))/n_a(z_a)$$

since $n_a(z_a)$ is chosen to minimize the per capita cost of producing z_a in a single-product club. Analogously, we know that

$$C(0,z_b,n(z_a,z_b))/n(z_a,z_b) \geq C(0,z_b,n_b(n_b))/n_b(z_b).$$

Thus, multiproduct clubs have gains from economies of scope as well as losses from imposing a common group size for the two public goods, making the direction of the inequality in (11) uncertain. Whether multiproduct clubs are superior to single-product clubs given the public good levels (z_a,z_b) depends on the magnitude of economies of scope and the difference in the congestion properties of the two public goods.

It remains to determine the optimal club structure when z_a and z_b are also chosen optimally. A partial answer to this question is formalized in the following proposition:

> **Proposition 1.** Let $n_a^* \equiv n_a(z_a^{*s})$, $n_b^* \equiv n_b(z_b^{*s})$, and $n^* \equiv n(z_a^{*m},z_b^{*m})$ denote optimal club sizes when z_a and z_b are chosen optimally under single-product and multiproduct clubs. Then, multiproduct clubs are uniquely optimal if
>
> $$C(z_a^{*s},z_b^{*s},n(z_a^{*s},z_b^{*s}))/n(z_a^{*s},z_b^{*s}) <$$
> $$C(z_a^{*s},0,n_a^*)/n_a^* + C(0,z_b^{*s},n_b^*)/n_b^* \qquad (12)$$

and single-product clubs are uniquely optimal if

$$C(z_a^{*m}, z_b^{*m}, n^*)/n^* >$$
$$C(z_a^{*m}, 0, n_a(z_a^{*m}))/n_a(z_a^{*m}) + C(0, z_b^{*m}, n_b(z_b^{*m}))/n_b(z_b^{*m}). \quad (13)$$

The first part of this proposition says that if the z levels that are optimal in single-product clubs can be provided at a lower per capita cost in a multiproduct club whose size is chosen optimally conditional on these z's, then multiproduct clubs are optimal. The second part of the proposition is a parallel statement for single-product clubs. To prove the first part of the proposition, note that, from (8) and (9), x^m and x^s can be written $x^m(z_a, z_b, n)$ and $x^s(z_a, z_b, n_a, n_b)$. Then, using (10), (12) implies that

$$x^{*s} \equiv x^s(z_a^{*s}, z_b^{*s}, n_a^*, n_b^*) < x^m(z_a^{*s}, z_b^{*s}, n(z_a^{*s}, z_b^{*s})). \quad (14)$$

It follows that

$$u^{*s} \equiv U[z_a^{*s}, z_b^{*s}, x^{*s}] < U[z_a^{*s}, z_b^{*s}, x^m(z_a^{*s}, z_b^{*s}, n(z_a^{*s}, z_b^{*s})]$$
$$\leq U[z_a^{*m}, z_b^{*m}, x^{*m}] \equiv u^{*m}, \quad (15)$$

where $x^{*m} \equiv x^m(z_a^{*m}, z_b^{*m}, n^*)$. A similar argument shows that $u^{*s} > u^{*m}$ when (13) holds. This proves the proposition.

While Proposition 1 is useful, it requires knowledge of the optimal public good levels, (z_a^{*s}, z_b^{*s}) and (z_a^{*m}, z_b^{*m}). It is clearly desirable to make a statement that does not depend on the location of the optimal z values:

Corollary 1. Multiproduct clubs are uniquely optimal if

$$C(z_a, z_b, n(z_a, z_b))/n(z_a, z_b) <$$
$$C(z_a, 0, n_a(z_a))/n_a(z_a) + C(0, z_b, n_b(z_b))/n_b(z_b) \quad (16)$$

for all (z_a, z_b) and single-product clubs are uniquely optimal if the inequality (16) is reversed for all (z_a, z_b).

To prove the first part of this corollary, note that if (16) holds for all (z_a, z_b), then it holds at $(z_a, z_b) = (z_a^{*s}, z_b^{*s})$. This implies that the condition (12) in Proposition 1 holds, from which it follows that $u^{*m} > u^{*s}$. Similarly, if the reverse of (16) holds for all (z_a, z_b), then it also holds for (z_a^{*m}, z_b^{*m}). This implies that the condition (13) in Proposition 1 holds, yielding $u^{*s} > u^{*m}$.

As is shown above, formation of multiproduct clubs leads to an efficiency loss because consumption group sizes cannot be adjusted according to the congestion properties of the individual public goods. When the congestion properties (and hence optimal group sizes) of the goods are identical, this loss disappears and multiproduct clubs are optimal. This notion is made precise in the following corollary:

Corollary 2. If $n_a^* = n_b^* = n^{**}$ and there exist economies of scope at (z_a^{*s}, z_b^{*s}) given a group size n^{**}, then multiproduct clubs are uniquely optimal.

To prove this result, note that the presence of economies of scope at (z_a^{*s}, z_b^{*s}) given a group size n^{**} means that

$$C(z_a^{*s}, z_b^{*s}, n^{**})/n^{**} < C(z_a^{*s}, 0, n^{**})/n^{**} + C(0, z_b^{*s}, n^{**})/n^{**}. \qquad (17)$$

But, since n^{**} does not necessarily minimize per capita cost in multiproduct clubs, it follows that

$$C(z_a^{*s}, z_b^{*s}, n(z_a^{*s}, z_b^{*s}))/n(z_a^{*s}, z_b^{*s}) \le C(z_a^{*s}, z_b^{*s}, n^{**})/n^{**}. \qquad (18)$$

Inequalities (17) and (18) imply that condition (12) of Proposition 1 holds, establishing Corollary 2.

To better understand the trade-off involved in multiproduct clubs, it is

useful to interpret the conditions stated in the proposition and the corollaries more intuitively. To do this, we define the degree of economies of scope and the efficiency loss from imposing a common group size as follows. The degree of economies of scope at output vector (z_a,z_b) given a group size n is defined as

$$h(z_a,z_b,n) \equiv [C(z_a,0,n) + C(0,z_b,n) - C(z_a,z_b,n)]/C(z_a,z_b,n), \qquad (19)$$

The degree of economies of scope thus measures the relative increase in cost that results from separating the production of (z_a,z_b) into two single-product clubs holding n fixed. Let $H(z_a,z_b) \equiv h(z_a,z_b,n(z_a,z_b))$ represent the degree of economies of scope at the optimal group size for a multiproduct club. Next, we define the efficiency loss in multiproduct clubs as

$$A(z_a,z_b) \equiv [\{C(z_a,0,n)/n - C(z_a,0,n_a)/n_a\} + \{C(0,z_b,n)/n - C(0,z_b,n_b)/n_b\}]/[C(z_a,0,n_a)/n_a + C(0,z_b,n_b)/n_b], \qquad (20)$$

where $n \equiv n(z_a,z_b)$, $n_a \equiv n_a(z_a)$, and $n_b \equiv n_b(z_b)$. The function A gives the relative increase in cost that results from imposing the optimal multiproduct group size on single-product clubs (note that $A(z_a,z_b) \geq 0$). Using (19) and (20), it is easily seen that if $H(z_a,z_b) > A(z_a,z_b)$, implying that the relative gain from economies of scope exceeds the relative loss from sacrificing separate consumption group sizes, then the first inequality in (11) holds. It follows that multiproduct clubs are superior given (z_a,z_b) in this case. Satisfaction of the reverse inequality means, of course, that single-product clubs are optimal.

The trade-off discussed above is shown in Figure 1 for a given (z_a,z_b). The distances A and B are the minimum per capita costs of producing z_a and z_b

in single-product clubs. Analogously, the distance M is the minimum per capita cost of producing z_a and z_b jointly in a multiproduct club. Our question is whether A + B > (≤) M. To put this inequality in a more useful form, note that the distance $A' - A$ is the efficiency loss from sacrificing the optimal consumption group size for public good z_a. That is, $A' - A = C(z_a,0,n)/n - C(z_a,0,n_a)/n_a$. Similarly, the distance $B' - B$ is the efficiency loss for z_b. The distance $A' + B' - M$ is the cost saving resulting from economies of scope ($A' + B' - M = C(z_a,0,n)/n + C(0,z_b,n)/n - C(z_a,z_b,n)/n$). Multiproduct clubs will then be optimal for given (z_a,z_b) when this cost saving exceeds the efficiency losses or when $A' + B' - M > A' - A + B' - B$. This inequality, of course, reduces to $A + B > M$. Note that although Figure 1 illustrates the case where $n(z_a,z_b)$ is located between $n_a(z_a)$ and $n_b(z_b)$, the general relationship among the optimal n's is indeterminate.

4. A Special Parameterization

If we are willing to assume that economies of scope can be summarized by a single parameter in the cost function, then more powerful results than those derived in section 3 are available. In particular, suppose that the cost function can be written $C(z_a,z_b,n,\sigma)$, where σ is the (nonnegative) complementarity parameter. A zero value of σ indicates the absence of economies of scope, so that

$$C(z_a,z_b,n,0) \equiv C(z_a,0,n,0) + C(0,z_b,n,0) \tag{21}$$

In addition, C is assumed to satisfy

$$C_\sigma(z_a,z_b,n,\sigma) < 0 \text{ if } z_a,z_b > 0 \tag{22}$$

$$C_\sigma(z_a,z_b,n,\sigma) = 0 \text{ if } z_a = 0 \text{ or } z_b = 0. \tag{23}$$

Note that when either of the z's is zero, as in (23), complementarity is not operative and an increase in σ has no effect on cost. Together, (21)-(23) imply that

$$C(z_a,z_b,n,\sigma) < C(z_a,0,n,\sigma) + C(0,z_b,n,\sigma) \tag{24}$$

for $\sigma > 0$, indicating the presence of economies of scope. Furthermore, it is easily seen that the degree of economies of scope from (19), rewritten as $h(z_a,z_b,n,\sigma)$ to include the complementarity parameter, is an increasing function of σ. Thus, the size of σ is a direct indicator of the strength of economies of scope as measured by h.

Since costs are independent of σ in single-product clubs by (23), it follows that the optimal single-product group sizes n_a^* and n_b^* are also independent of σ. Bearing this in mind, the following result can be established:

> **Proposition 2.** There exists some critical σ value $\sigma^* \geq 0$ such that single-product clubs are optimal for $0 \leq \sigma < \sigma^*$ and multiproduct clubs are optimal for $\sigma > \sigma^*$. If $n_a^* = n_b^*$, then $\sigma^* = 0$. Otherwise, $\sigma^* > 0$.

This proposition says that there is some critical magnitude of economies of scope beyond which the gains from cost complementarity outweigh the efficiency loss from a common group size, making multiproduct clubs optimal. When the scope effect is weaker than this critical value, the efficiency loss dominates and single-product clubs are optimal. The proposition is proved by a simple application of the envelope theorem to the Lagrangean expressions of the single-product and multiproduct club optimization problems. Viewing the optimal values of the choice variables in the two problems as functions of σ,

the utility differential between multiproduct clubs and single-product clubs can be written

$$D(\sigma) \equiv U(z_a^{*m}, z_b^{*m}, x^{*m}) - U(z_a^{*s}, z_b^{*s}, x^{*s}). \qquad (25)$$

Applying the envelope theorem, it follows that

$$D'(\sigma) = \mu(N/n^*)C_\sigma(z_a^{*m}, z_b^{*m}, n^*, \sigma) > 0, \qquad (26)$$

where μ is the negative multiplier associated with the constraint (2). Eq. (26) says that the difference between the maximal utilities in multiproduct and single-product clubs is an increasing function of the complementarity parameter σ. In addition to using (22), this result invokes the assumption (23) that the costs of single-product clubs are unaffected by an increase in σ (this yields $dU(z_a^{*s}, z_b^{*s}, x^{*s})/d\sigma = 0$). The next step in the proof is to note that if $n_a^* = n_b^*$, then the maximal utilities are the same in both types of clubs when $\sigma = 0$. This follows because when economies of scope are absent and (21) holds, the multiproduct club constraint (2) can be derived from the single-product club constraint (3) by imposing the side condition $n_a = n_b$. If this side condition is satisfied at the single-product club solution, it follows that its imposition does not change the value of the objective function, implying that the maximal utilities are the same for both types of clubs ($D(0) = 0$). If, on the other hand, $n_a^* \neq n_b^*$, then imposition of the side condition has an effect and the single-product and multiproduct solutions differ, with the multiproduct club utility being lower ($D(0) < 0$). Combining the above results, we see that since $D'(\sigma) > 0$ and since $D(0) = 0$ when $n_a^* = n_b^*$, multiproduct clubs are optimal ($D(\sigma) > 0$) for all $\sigma > 0$, implying that the critical value σ^* equals zero. On the other hand, since $D(0) < 0$ when $n_a^* \neq n_b^*$ and $D'(\sigma) > 0$, there

exists some $\sigma^* > 0$ such that single-product clubs are optimal ($D(\sigma) < 0$) for all σ below σ^* and multiproduct clubs are optimal ($D(\sigma) > 0$) for all σ above σ^*. This proves the proposition. Note finally that in the case where $n_a^* = n_b^*$, Proposition 2 is a restatement of Corollary 2 from above for the special parameterization.

5. Example

This section presents a concrete example based on the parameterization of section 4. Since computation of σ^* is not possible without specification of the utility function, we instead take the approach of section 3, analysing the choice between club types by comparing costs. Suppose that the cost function is given by

$$C(z_a, z_b, n) = \alpha_a z_a + \beta_a z_a n^2 + \alpha_b z_b + \beta_b z_b n^2 - \sigma z_a z_b, \tag{27}$$

where α_i, $\beta_i > 0$, $i = a,b$. Note that this function is not always well-defined since cost can be negative for given z values when σ is large (or when the z's are large for a given σ). This fact will be taken into account below. In this case, the per capita cost-minimization conditions yield

$$n_a(z_a) = (\alpha_a/\beta_a)^{1/2} \tag{28}$$

$$n_b(z_b) = (\alpha_b/\beta_b)^{1/2} \tag{29}$$

$$n(z_a, z_b) = [(\alpha_a z_a + \alpha_b z_b - \sigma z_a z_b)/(\beta_a z_a + \beta_b z_b)]^{1/2}. \tag{30}$$

Note that the optimal n's for single-product clubs are independent of the z's. Substituting (28)-(30) into (27) yields

$$C(z_a, 0, n_a(z_a))/n_a(z_a) = 2z_a(\alpha_a\beta_a)^{1/2} \tag{31}$$

$$C(0,z_b,n_b(z_b))/n_b(z_b) = 2z_b(\alpha_b\beta_b)^{1/2} \tag{32}$$

$$C(z_a,z_b,n(z_a,z_b))/n(z_a,z_b) = 2[(\beta_a z_a + \beta_b z_b)(\alpha_a z_a + \alpha_b z_b - \sigma z_a z_b)]^{1/2} \tag{33}$$

Recalling the approach of section 3, our goal is to determine whether C_a/n_a + C_b/n_b from (31) and (32) is greater or less than C/n from (33). Squaring both expressions and simplifying, it is easily seen that, for given (z_a,z_b), single-product clubs cost more (less) than multiproduct clubs as

$$2(\alpha_a\beta_a\alpha_b\beta_b)^{1/2} > (<)\ \alpha_b\beta_a + \alpha_a\beta_b - \sigma(\beta_a z_a + \beta_b z_b). \tag{34}$$

The key to evaluating (34) is to use (28) and (29) and note that $\alpha_b\beta_a + \alpha_a\beta_b$ equals the term on the LHS when $n_a(z_a) = n_b(z_b)$ and exceeds this term otherwise. This follows because squaring the two expressions and subtracting yields $(\alpha_b\beta_a + \alpha_a\beta_b)^2 - 4\alpha_a\beta_a\alpha_b\beta_b = (\alpha_b\beta_a - \alpha_a\beta_b)^2$, which is zero when $\alpha_a/\beta_a = \alpha_b/\beta_b$ and positive otherwise. An implication of this fact is that the LHS of (34) will be greater than the RHS (indicating the superiority of multiproduct clubs for given (z_a,z_b)) when $n_a(z_a) = n_b(z_b)$ and $\sigma > 0$.

It is clear from above that the direction of the inequality in (34) is uncertain when $\alpha_a/\beta_a \neq \alpha_b/\beta_b$. However, noting that the optimal public good levels in single-product clubs (z_i^{*s}, i = a,b) are independent of σ, the first inequality in (34) will hold at z_a^{*s} and z_b^{*s} as long as

$$\sigma > [(\alpha_b\beta_a + \alpha_a\beta_b) - 2(\alpha_a\beta_a\alpha_b\beta_b)^{1/2}]/(\beta_a z_a^{*s} + \beta_b z_b^{*s}). \tag{35}$$

Since satisfaction of (35) means that inequality (12) of Proposition 1 holds, it follows that multiproduct clubs are optimal when the complementarity parameter σ is sufficiently large. Note that while the result is similar, this is not the same conclusion as in Proposition 2 since it is not true that

single-product clubs are optimal for σ less than the expression in (35). It also should be noted that an upper bound must be placed on σ given that $n(z_a,z_b)$ from (30) is not well-defined for σ arbitrarily large. For (30) (and hence (35)) to make sense at (z_a^{*s},z_b^{*s}), σ must satisfy the inequality

$$\sigma < (\alpha_a z_a^{*s} + \alpha_b z_b^{*s})/z_a^{*s} z_b^{*s}. \tag{36}$$

Thus, for the optimality of multiproduct clubs to be guaranteed, σ must satisfy (36) in addition to (35) (since the RHS of (35) is less than the RHS of (36), such σ's exist).

6. The Equilibrium Club Configuration

The purpose of the analysis in this section of the paper is to characterize the equilibrium club configuration when clubs are organized by competitive developers. The analysis, which parallels that of Berglas (1976), is not meant to be realistic since developer-provided public goods are rare in the real world. Rather, the purpose of the discussion is to see whether the optimal club configuration emerges from decentralized behavior in an idealized competitive world. The analysis first derives the features of single-product and multiproduct clubs organized by developers. The discussion then identifies the club configuration (single-product or multiproduct) that actually emerges in equilibrium.

Consider the problem faced by a developer organizing a multiproduct club. The developer charges a club entry fee denoted by P while choosing the public good levels in the club and the size of its population. Suppose that the developer wishes to guarantee a utility level u to his club members. Then P must satisfy

$$U(z_a, z_b, I-P) = u. \tag{37}$$

The developer's goal is to choose z_a, z_b, P, and n to maximize the profit expression

$$\pi = nP - C(z_a, z_b, n) \tag{38}$$

subject to (37). It is easy to show that the first-order conditions for this problem are the Samuelson conditions (4) and $P - C_3 = 0$. The profit level realized by the developer depends on the utility level u. Profit π can be shown to be a decreasing function of u,[4] and it can be shown that when $u = u^{*m}$, $\pi = 0$. These facts imply that $\pi \geq (<) \ 0$ as $u \leq (>) \ u^{*m}$. To see that profit is zero when $u = u^{*m}$, note that $P = C/n$ holds when $\pi = 0$, so that the condition $P - C_3 = 0$ reduces to the per capita cost-minimization condition (5). Since the Samuelson conditions also hold and since the zero profit condition together with the budget constraint $x + P = I$ implies satisfaction of the club resource constraint $nx + C = nI$, it follows that the planning conditions and club equilibrium conditions are the same. This means that the u value associated with a zero profit equals u^{*m}.

Now consider the problem of a type-a single-product club developer, who charges an entry fee P_a while choosing the public good level z_a in the club as well as the size n_a of its population. In setting the levels of these variables, the developer views the characteristics of type-b clubs (which are chosen by other developers) as parametric. In this sense, the equilibrium under consideration is of the Nash variety. If the developer wishes to guarantee a utility of u_a to his club members, then z_a and P_a must satisfy

$$U(z_a, z_b, I-P_a-P_b) = u_a. \tag{39}$$

where z_b and P_b are the parametric public good level and entry fee for type-b clubs. The developer's goal is to choose z_a, P_a, and n_a to maximize profit

$$\pi_a = n_a P_a - C(z_a,0,n_a), \tag{40}$$

subject to (39). The first-order conditions again yield the appropriate Samuelson condition (in this case (6)) and $P_a - C_{a3} = 0$.

Parallel analysis applies to the developers of type-b single-product clubs. Viewing z_a and P_a as parametric, type-b developers choose z_b, P_b, and n_b to maximize profit π_b while guaranteeing a utility of u_b to club members. In equilibrium, the utility goals of developers should be consistent, so that $u_a = u_b = u$, and profit should be the same for both types of clubs, so that $\pi_a = \pi_b = \pi$. For given u, the Nash solution for z_i, P_i, and n_i, $i = a,b$ is then determined by the four first-order conditions, the utility constraint, and the equal-profit condition. It can be shown that the (common) profit level is a decreasing function of the utility level u and that profit is zero when $u = u^{*s}$.[5]

With the above background, we derive the club configuration that emerges in equilibrium. Equilibrium is defined as follows:

Definition 2. An equilibrium of the model is a utility level u_E and an associated club configuration such that i) developers earn nonnegative profit and ii) any club configuration offering $u > u_E$ has negative profit.

Under this definition, the following result can be established:

Proposition 3. When $u^{*m} > u^{*s}$, the equilibrium club configuration consists of multiproduct clubs offering $u_E = u^{*m}$. When the reverse inequality holds, the equilibrium club configuration consists of single-product clubs offering $u_E = u^{*s}$.

This proposition states the important result that equilibrium in the model is efficient. To prove this result, first note that any multiproduct club configuration offering $u \neq u^{*m}$ cannot be an equilibrium. From above, if $u > u^{*m}$ then multiproduct clubs lose money. While profits are positive when $u = u' < u^{*m}$, such a club configuration also cannot be an equilibrium because an alternative multiproduct club configuration offering a u value between u' and u^{*m} gives a higher utility and earns positive profit. A parallel argument shows that any single-product club configuration offering $u \neq u^{*s}$ cannot be an equilibrium.

With the class of potential equilibrium club configurations thus narrowed to the zero-profit configurations, Proposition 3 follows easily. In the case where $u^{*m} > u^{*s}$, the single-product club configuration cannot be the equilibrium because there exist alternative multiproduct club configurations offering u between u^{*m} and u^{*s} that earn positive profit. Similarly, when $u^{*s} > u^{*m}$, the multiproduct club configuration cannot be the equilibrium because there exist alternative single-product club configurations offering u between u^{*s} and u^{*m} that earn positive profit. This establishes the proposition.[6]

7. Conclusion

This paper has drawn a connection between club theory and recent research in industrial organization by analysing the optimality of multiproduct clubs in the presence of economies of scope. The paper has shown that in deciding between single-product and multiproduct clubs, the planner must compare the cost savings from economies of scope to the efficiency loss from imposing a common consumption group size on different public goods. While the equilibrium analysis shows that the correct club structure emerges from the decentralized actions of competitive developers, it is an open question whether public good

provision is organized efficiently in the real world. However, the fact that public goods with obvious cost complementarities (such as police and fire protection) are provided jointly while other unrelated goods such as parks are often provided in separate jurisdictions is an encouraging sign. This pattern suggests that even in the absence of a truly competitive environment for the provision of local public goods, something close to an optimal jurisdiction structure may emerge in the real world.

Table 1

Local Governmental Units in the U.S.*

	1952	1982
County	3052	3041
Municipal	16807	19076
School Districts	67355	14851
Special Districts	12340	28588

*Source: 1982 Census of Governments

Table 2

Single-Function Special Districts in the U.S. (1982)*

Type	Number
Library	638
Health and Hospital	1226
Highway	598
Airport	357
Fire	4560
Drainage and Flood Control	2705
Soil and Water Conservation	2409
Parks and Recreation	924
Housing and Community Development	3296
Sewerage	1631
Water Supply	2637

*Source: 1982 Census of Governments. Some district types are omitted.

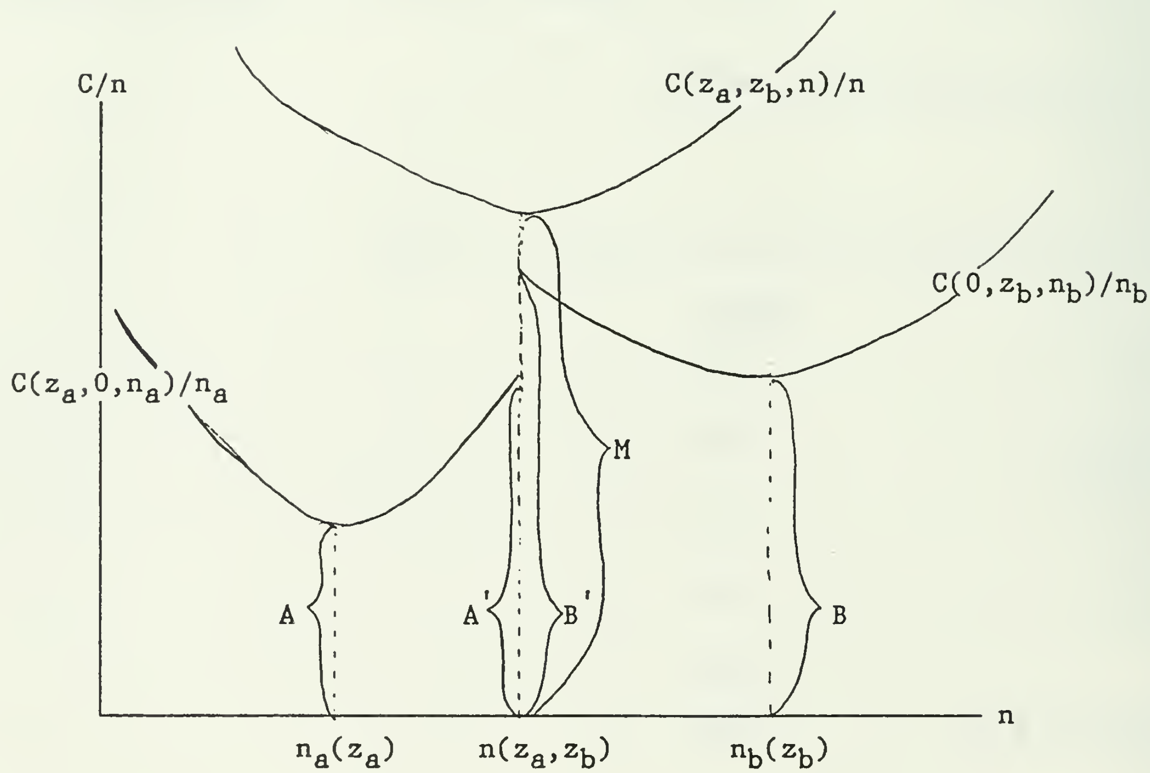

Figure 1

REFERENCES

Baumol, W., Panzar, J. and Willig. R. (1982). Contestable Markets and The Theory of Industry Structure. New York: Harcourt Brace Jovanovich.

Berglas, E. (1976). On the theory of clubs. American Economic Review, 66, 116-121.

Berglas, E. and Pines, D. (1981). Clubs, local public goods and transportation models: a synthesis. Journal of Public Economics, **15**, 141-162.

International City Management Association (1977). Police/fire consolidation in municipalities 10,000 and over. Urban Data Service, **9**, 1-9.

Sandler, T. and Tschirhart, J. (1980). The economic theory of clubs: an evaluative survey. Journal of Economic Literature, **18**, 1481-1521.

U.S. Department of Commerce, Bureau of the Census (1983). 1982 Census of Governments. Washington D.C.: U.S. Government Printing Office.

NOTES

* Professor of Economics and Ph.D. candidate in Economics respectively.

1. Township governments are omitted.

2. See International City Management Association (1977) for a discussion of these gains.

3. It is worth noting that in their survey paper, Sandler and Tschirhart (1980) suggest that the theory of the multiproduct firm could be used to analyse multiproduct clubs.

4. This follows from a simple application of the envelope theorem.

5. The first claim can be proved by noting that the Nash solution is the same as the solution for the problem of maximizing π_a subject to (39) and $\pi_a = \pi_b$. Applying the envelope theorem to this problem shows that π_a is decresing in u. The second claim is established in a manner parallel to the analogous claim above.

6. It should be noted that the above equilibrium concept can be criticized for its notion of deviations away from the equilibrium. In an ideal definition, a club configuration would fail to be an equilibrium when an isolated developer could set up a profitable alternative club that would attract residents and earn him a positive profit. Unfortunately, our characterization of deviations away from a multiproduct club configuration does not conform to such a definition (deviations away from a single-product club configuration do conform, however). The problem is that, given the Nash character of the single-product club developer's optimization problem, interaction between developers is inherent in the single-product club configuration. As a result, deviation away from a multiproduct club configuration by an <u>isolated</u> single-product club developer is not a well-defined action. To overcome this problem, we posit that a club configuration is not an equilibrium when there exists an alternative club <u>configuration</u> (rather than a single alternative club set up by an isolated developer) that offers a higher utility and leads to nonnegative profit. For this reason, equilibrium under our definition is not strictly competitive.

Papers in the Political Economy of Institutions Series

No. 1 Susan I. Cohen. "Pareto Optimality and Bidding for Contracts." Working Paper # 1411

No. 2 Jan K. Brueckner and Kangoh Lee. "Spatially-Limited Altruism, Mixed Clubs, and Local Income Redistribution." Working Paper #1406

No. 3 George E. Monahan and Vijay K. Vemuri. "Monotonicity of Second-Best Optimal Contracts." Working Paper #1417

No. 4 Charles D. Kolstad, Gary V. Johnson, and Thomas S. Ulen. "Ex Post Liability for Harm vs. Ex Ante Safety Regulation: Substitutes or Complements?" Working Paper #1419

No. 5 Lanny Arvan and Hadi S. Esfahani. "A Model of Efficiency Wages as a Signal of Firm Value." Working Paper #1424

No. 6 Kalyan Chatterjee and Larry Samuelson. "Perfect Equilibria in Simultaneous-Offers Bargaining." Working Paper #1425

No. 7 Jan K. Brueckner and Kangoh Lee. "Economies of Scope and Multiproduct Clubs." Working Paper #1428